Middlemen in Peer-to-Peer Networks: Stability and Efficiency*

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Abstract

A fundamental tension of between the efficiency and stability of social networks is now a well recognized fact in the literature. In general, networks are not simultaneously efficient and stable. Here we consider strongly pairwise stable networks and identify that the presence of middlemen is of particular importance for such a network to be efficient as well. We find that for the component wise egalitarian rule there is no conflict between the efficient and stable networks when these middlemen have no incentive to break up the network.

Keywords: Networks; pairwise stability; critical link; middleman.

JEL Classification: C71, C72.

1 Introduction

Jackson and Wolinsky (1996) put forward the fundamental insight that there is a profound tension between efficiency and stability in game theoretic models of network formation. Indeed, networks that generate maximal collective values — indicated as *efficient networks* — are usually not stable in the sense that players have incentives to delete existing links or create new links. Since their seminal contribution, many papers have examined this fundamental tension between efficiency and stability of social networks in different settings. For a discussion of this literature we refer to the excellent review by Jackson (2003).

Jackson and Wolinsky (1996) also discuss an exception to the rule that efficiency and stability are conflicting properties in the context of social networks. They showed that under component-wise egalitarian payoffs there might exist efficient networks that are *pairwise stable*. That is, there might exist networks that generate maximal collective value, in which any participating player does not have incentives to delete a single link and any pair of players does not have incentives to form an additional link.

In this paper we investigate whether this result can be extended to include networks that are stable with respect to a more reasonable link-based stability concept called "strong pairwise stability".

Pairwise stability is a concept that suffers from a serious deficiency. The hypothesis that individual players only consider the deletion and creation of a single link seems unrealistic. Clearly individual players can and will consider the deletion of one *or more* links under her control in the network, particularly since link deletion involves unilateral action. This is formulated in the concept of strong pairwise stability introduced by Gilles and Sarangi (2004).[1]

This concept incorporates the possibility that individual players might delete multiple links while any pair of players considers the creation of an additional link. It is based on the principle that individuals have complete control over the existence of links in which they participate. Hence, any player can unilaterally delete any set of links in which she participates. Similarly, the creation of a link requires the consent of both players involved.

*This paper summarizes part of the research presented in Gilles, Chakrabarti, Sarangi and Badasyan (2004). For all proofs and extended discussions we refer to that paper. We are grateful for the helpful remarks and suggestions made by three anonymous reviewers of the Harvard Peer-to-Peer Workshop on a previous draft of this paper.

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[1] We remark that Jackson and Wolinsky (1996) already indicated, without formalizing, several generalizations of their pairwise stability concept, including what we call strong pairwise stability in this paper. Bloch and Jackson (2004) also use the notion of strong pairwise stability, but label it as pairwise stability*. Closely related to this is also the notion of pairwise stable equilibrium studied by Goyal and Joshi (2003).

It can be shown that for certain normal form game-theoretic models of network formation, Nash equilibria are characterized by stability against the removal of sets of links by individual players. (Gilles and Sarangi 2004, Propositions 3.1 and 3.10) This is also recognized by Goyal and Joshi (2003) and Bloch and Jackson (2004) who discuss pairwise stable equilibrium networks. This concept combines the Nash equilibrium property with stability against pairs of players forming additional links. This concept is therefore closely related to strong pairwise stability.[2]

In the present paper we show that the coincidence of efficiency and stability can indeed be extended to strong pairwise stability for component-wise egalitarian payoffs. Jackson and Wolinsky (1996) showed that *critical links* have to be neutralized in order to establish pairwise stable and efficient networks. Here we establish that middlemen in the network must not have any incentives to break communication in the network thereby making the network secure against deviations by such middlemen. A middleman occupies a critical position in that she can disrupt communication lines in the network by removing certain links under her control.

The shift in the focus from critical links to middleman positions in a network is very natural outcome of our stability concept. Moreover it focusses the analysis on the role of middlemen who play a central role in the functioning of many social networks. This poses important questions regarding the role and power of such middlemen in the processes that lead to network formation. Power of middlemen were only investigated for the core in a three person trade economy by Kalai, Postlewaite, and Roberts (1978). Future research should investigate the power of middlemen further in the context of general payoff functions and network formation principles based on link formation with mutual consent.

The rest of the paper is organized as follows. Sections 2 and 3 introduce notation and various stability notions mentioned above. Section 4 summarizes the main insight from Jackson and Wolinsky (1996) on critical links and the possibility of efficient and pairwise stable networks. Section 5 extends this insight to strong pairwise stability, using the notion of middleman security. Section 6 concludes.

2 Networks and values

Throughout we let $N = \{1, 2, \dots, n\}$ be a finite set of players. Two distinct players $i, j \in N$ with $i \neq j$ are *linked* if i and j are related in some (social) capacity. These relationships are *undirected* in the sense that the two players forming a relationship are equals within that relationship. Formally, an (undirected) link between i and j is defined as the set $\{i, j\}$. Throughout we use the shorthand notation ij to denote the link $\{i, j\}$. It should be clear that ij is completely equivalent to ji.

In total there are $\frac{1}{2}n(n-1)$ potential links on the player set N. The collection of all potential links on N is denoted by

$$g_N = \{ij \mid i, j \in N \text{ and } i \neq j\} \tag{1}$$

A *network* g is now defined as any collection of links $g \subset g_N$. The collection of all networks on N is denoted by $\mathbb{G}^N = \{g \mid g \subset g_N\}$. The collection $\mathbb{G}^N$ consists of $2^{\frac{1}{2}n(n-1)}$ networks.

For every network $g \in \mathbb{G}^N$ and every player $i \in N$ we denote i's *neighborhood* in g by $N_i(g) = \{j \in N \mid j \neq i \text{ and } ij \in g\}$. Player i therefore is participating in the links in her *link set* $L_i(g) = \{ij \in g \mid j \in N_i(g)\} \subset g$. We also define $N(g) = \cup_{i \in N} N_i(g)$ and let $n(g) = \# N(g)$ with the convention that if $N(g) = \varnothing$, we let $n(g) = 1$.[3]

Let $\pi \colon N \to N$ be a permutation on N. For every network $g \in \mathbb{G}^N$ the corresponding permutation is denoted by $g^\pi = \{\pi(i)\pi(j) \mid ij \in g\} \in \mathbb{G}^N$.

2.1 Paths and network components

A *path* in g connecting i and j is a set of distinct players $\{i_1, i_2, \dots, i_p\} \subset N(g)$ with $p \geqslant 2$ such that $i_1 = i$, $i_p = j$, and $\{i_1 i_2, i_2 i_3, \dots, i_{p-1} i_p\} \subset g$. The network $h \subset g$ is a *component* of g if for all $i \in N(h)$ and $j \in N(h)$, $i \neq j$, there exists a path in h connecting i and j and for any $i \in N(h)$ and $j \in N(g)$, $ij \in g$ implies $ij \in h$. In other words, a component is a maximally connected subnetwork of g. We denote the class of network components of the network g by $C(g)$.

The set of players that are (fully) *disconnected* in the network g is denoted by

$$N_0(g) = N \setminus N(g) = \{i \in N \mid N_i(g) = \varnothing\}. \tag{2}$$

Furthermore, we define

$$\Gamma(g) = \{N(h) \mid h \in C(g)\} \cup \{\{i\} \mid i \in N_0(g)\} \tag{3}$$

as the partitioning of the player set N based on the component structure of the network g.[4]

2.2 Network values

With regard to the description of benefits or "utilities" generated by participation in a network, we limit our discussion in this paper to so-called *collective network benefit functions*

[2] For a formal analysis of the relationship between pairwise stable equilibrium and strong pairwise stability we refer to Bloch and Jackson (2004).

[3] We emphasize here that if $N(g) \neq \varnothing$, we have that $n(g) \geqslant 2$. Namely, in those cases the network has to consist of at least one link.

[4] We therefore distinguish a link-based partitioning of a network g into components, denoted by $C(g)$, from a node-based partitioning, denoted by $\Gamma(g)$. Our analysis calls for both partitioning conventions to be used throughout.

given by $v\colon \mathbb{G}^N \to \mathbb{R}$ such that $v(\varnothing) = 0$. Following Jackson and Wolinsky (1996), we denote such functions as "network value" functions. A network value function v assigns a total benefit $v(g) \in \mathbb{R}$ to the network $g \in \mathbb{G}^N$. The space of all network value functions v such that $v(\varnothing) = 0$ is denoted by $\mathbb{V}^N$.

Let $v \in \mathbb{V}^N$ be some network value function. We consider two fundamental properties of such a network value function:

(i) The network value function v is *component additive* if $v(g) = \sum_{h \in C(g)} v(h)$.

(ii) The network value function v is *anonymous* if $v(g^\pi) = v(g)$ for all permutations π and networks g.

Finally, we define an efficiency concept. A network $g \in \mathbb{G}^N$ is *efficient* with respect to value function v if $v(g) \geqslant v(g')$ for all $g' \subset g$.[5]

2.3 Allocation rules

Following the literature, we consider the allocation of network values over all players in a network. The allocated payoff to an individual player is determined by an *allocation rule* $Y\colon \mathbb{G}^N \times \mathbb{V}^N \to \mathbb{R}^N$ which determines how for any network $g \in \mathbb{G}^N$ the collective value $v(g)$ is distributed over the players in N. $Y_i(g, v)$ is the payoff to player i from the network g under the value function v.

Let $\pi\colon N \to N$ be a permutation. Now v^π is defined by $v^\pi(g^\pi) = v(g)$.

(i) An allocation rule Y is *anonymous* if for any permutation π, $Y_{\pi(i)}(g^\pi, v^\pi) = Y_i(g, v)$. Anonymity of the allocation rule simply means that the payoff of a player depends solely on the position in the network rather than the label or name of that player.

(ii) An allocation rule Y is *balanced* if $\sum_{i \in N} Y_i(g, v) = v(g)$ for all v and g.[6]

(iii) An allocation rule Y is *component balanced* if $\sum_{i \in N(h)} Y_i(g, v) = v(h)$ for every g and $h \in C(g)$ and every component additive v. Component balance along with component additivity implies that fully disconnected players in $N_0(g)$ always have an allocated payoff of zero.

Let $v \in \mathbb{V}^N$. The *component-wise egalitarian allocation rule* is defined by

$$Y_i^{ce}(g, v) = \frac{v(h_i)}{n(h_i)} \tag{4}$$

where $h_i \in C(g)$ such that $i \in N(h_i)$ and $h_i = \varnothing$ if there is no $h \in C(g)$ such that $i \in N(h)$. Under this allocation rule, the value generated by a component is split equally among the members of that component.

It is clear that Y^{ce} is the *unique* allocation rule Y that is component balanced and assigns an equal payoff to all players in the same component of a network, i.e., for all $(g, v) \in \mathbb{G}^N \times \mathbb{V}^N$ it holds that

$$Y_i(g, v) = Y_j(h, v) \tag{5}$$

for every $h \in C(g)$ and all $i, j \in N(h)$.

Finally we remark that $Y^{ce}(\cdot, v)$ is balanced for every component additive $v \in \mathbb{V}^N$. The component-wise egalitarian payoff rule is not balanced for arbitrary network value functions.

3 Stability properties

In this section we discuss network formation principles from a link-based perspective. Central to this approach is that the formation of each link in principle is considered separately. Since the formation of a link in the networks involves a pair of players, mutual consent is required. On the other hand, each player can delete an established link unilaterally.

Denote by $g + ij$ the network obtained by adding link ij to the existing network g, i.e., $g + ij = g \cup \{ij\}$. Similarly, $g - ij$ denotes the network that results from deleting link ij from the existing network g, i.e., $g - ij = g \setminus \{ij\}$.

Let Y be some allocation rule. We introduce three fundamental network stability properties that describe the network formation principles formulated above.

(i) A network $g \in \mathbb{G}^N$ is *link deletion proof* (LDP) under Y if for every player $i \in N$ and every neighbor $j \in N_i(g)$, it holds that $Y_i(g - ij, v) \leqslant Y_i(g, v)$. Link deletion proofness requires that each individual player has no incentive to sever an existing link with one of his neighbors.

(ii) A network $g \in \mathbb{G}^N$ is *strong link deletion proof* (SLDP) under Y if for every player $i \in N$ and every link set $h \subset L_i(g)$, it holds that $Y_i(g \setminus h, v) \leqslant Y_i(g, v)$. Strong link deletion proofness requires that each player has no incentive to sever links with one or more of his neighbors. Obviously, SLDP implies LDP.

(iii) A network $g \in \mathbb{G}^N$ is *link addition proof* (LAP) under Y if for all players $i, j \in N$, it holds that $Y_i(g + ij, v) > Y_i(g, v)$ implies $Y_j(g + ij, v) < Y_j(g, v)$. Link addition proofness states that there are no incentives to form additional links. This is founded on a process of mutual consent in link formation. Indeed, if one player

[5] Jackson and Wolinsky (1996) refer to these as strongly efficient networks, implicitly implying thereby that weak efficiency corresponds to a Paretian efficiency notion.

[6] Balance is also known as "efficiency" in the networks literature.

would like to add a link, the other player would have strong objections.[7]

Jackson and Wolinsky (1996) introduced link deletion proofness and link addition proofness, although they did not explicitly define these concepts as such. Strong link deletion proofness was introduced by Gilles and Sarangi (2004).

These three fundamental stability concepts can be used to define additional stability concepts. A network $g \in \mathbb{G}^N$ is *pairwise stable* under Y if it is LDP as well as LAP under Y. Furthermore, a network $g \in \mathbb{G}^N$ is *strongly pairwise stable* under Y if it is SLDP as well as LAP under Y.

The main difference between regular pairwise stability and strong pairwise stability is that individual players are allowed to remove multiple of their links rather than a single link. Pairwise stability was seminally developed by Jackson and Wolinsky (1996). Strong pairwise stability has been the subject of Gilles and Sarangi (2004) and Gilles, Chakrabarti, Sarangi, and Badasyan (2004). Bloch and Jackson (2004) investigate strong pairwise stability and the notion of pairwise stable equilibrium developed by Goyal and Joshi (2003), which in some cases is similar.

Next we show with the use of an example that pairwise stability has serious limitations in the sense that individual players are considered to have no power to delete multiple links even in situations in which this is extremely desirable.

Example 3.1 Being stuck with bad company
Consider a three player situation with $N = \{1, 2, 3\}$. For simplification of notation we denote the potential links in this situation as follows: $a = 12$, $b = 13$, and $c = 23$. Hence, $\mathbb{G}^N = \{\varnothing, a, b, c, ab, ac, bc, abc\}$.
Let $\alpha > 0$. We consider an allocation rule $\overline{Y}\colon \mathbb{G}^N \times \mathbb{V}^N \to \mathbb{R}$ which for every $v \in \mathbb{V}^N$ is defined by

$$\overline{Y}(\varnothing, v) = (0, 0, 0)$$
$$\overline{Y}(a, v) = \left(\tfrac{v(a)}{2}, \tfrac{v(a)}{2}, 0\right)$$
$$\overline{Y}(b, v) = \left(\tfrac{v(b)}{2}, 0, \tfrac{v(b)}{2}\right)$$
$$\overline{Y}(c, v) = \left(0, \tfrac{v(c)}{2}, \tfrac{v(c)}{2}\right)$$
$$\overline{Y}(ab, v) = (v(ab), 0, 0)$$
$$\overline{Y}(ac, v) = \left(-\alpha v(abc), v(ac) - \tfrac{1}{2}(1-\alpha)v(abc), \tfrac{1}{2}(1+\alpha)v(abc)\right)$$
$$\overline{Y}(bc, v) = \left(-\alpha v(abc), \tfrac{1}{2}(1+\alpha)v(abc), v(bc) - \tfrac{1}{2}(1-\alpha)v(abc)\right)$$
$$\overline{Y}(abc, v) = \left(-\alpha v(abc), \tfrac{1}{2}(1+\alpha)v(abc), \tfrac{1}{2}(1+\alpha)v(abc)\right)$$

Note that $\overline{Y}$ is component balanced. Our main claim is now that in general, under the allocation rule $\overline{Y}$, the complete network abc is LDP, but not SLDP:

Claim: *If $v \in \mathbb{V}^N$ such that $v(g) > 0$ for every $g \neq \varnothing$, then the network $g^\star = abc$ is link deletion proof, but not strong link deletion proof, with respect to the allocation rule $\overline{Y}$.*

Hence, without the possibility of a player to remove multiple of his links simultaneously, he might get stuck with "bad company". Indeed, here player 1 would like to remove his links with player 2 as well as player 3, but using LDP he can only remove at most one of these two links. Under SLDP player 1 is able to remove both links and improve his situation.
For a proof of the claim we refer to Gilles, Chakrabarti, Sarangi, and Badasyan (2004), Example 3.1. □

4 Critical links

In this section we explore the relationship between strongly pairwise stable and efficient networks under the component-wise egalitarian allocation rule, originally developed in Jackson and Wolinsky (1996). We first discuss the notion of a critical link.

Definition 4.1 *A link $ij \in g \in \mathbb{G}^N$ is **critical** in the network g if $\#\Gamma(g) < \#\Gamma(g - ij)$.*

Hence, a link is critical if after its removal either the number of components of the network increases, or the number of disconnected players increases. It means that there is no alternative path to replace such a critical link.

Let $h \in C(g)$ denote a component that contains a critical link in the network $g \in \mathbb{G}^N$ and let $h_1 \subset h$ and $h_2 \subset h$ denote components obtained from h by severing that critical link. (Note that it may be the case that $h_1 = \varnothing$ or $h_2 = \varnothing$.)

The following concept of critical link monotonicity has been introduced by Jackson and Wolinsky (1996) in their discussion of certain properties of the component-wise egalitarian allocation rule Y^{ce}.

Definition 4.2 *The pair (g, v) satisfies **critical link monotonicity** if for any critical link $ij \in h$ with $h \in C(g)$ and the two associated components h_1 and h_2 of $h - ij$, we have that*

$$v(h) \geqslant v(h_1) + v(h_2) \implies \frac{v(h)}{n(h)} \geqslant \max\left[\frac{v(h_1)}{n(h_1)}, \frac{v(h_2)}{n(h_2)}\right] \quad (6)$$

As shown by Jackson and Wolinsky (1996), this constitutes a necessary and sufficient condition for the existence of efficient networks that are pairwise stable with regard to the component egalitarian allocation rule:

[7] Considering one link at the time with regard to the formation of that link seems natural. A generalization to the simultaneous formation of multiple links would not yield much unless generalized to coalitional considerations. Such coalitional considerations are at the foundation of the notion of strong stability introduced and analyzed by Jackson and van den Nouweland (2004).

Lemma 4.3 (Jackson and Wolinsky 1996, Claim, page 61) *If g is efficient relative to a component additive v, then g is pairwise stable for Y^{ce} relative to v if and only if (g, v) satisfies critical link monotonicity.*

That payoffs are egalitarian imposes that the individual players have essentially the same objectives, namely maximizing the total value generated by the network in which they participate. Moreover, component additivity of the component-wise egalitarian allocation rule links this individual drive correctly to the component in which these individuals operate. The only players to be checked are those players participating in critical links; they are guaranteed to have the proper incentives through the property of critical link monotonicity.

5 Middleman positions

A critical link referred to a single link between two players, which removal resulted into a disintegration of the network. When a single player removes multiple links and the network disintegrates, then we call such a player a *middleman* in the network. In graph theory, the position of a middleman in the network is also referred to as a "cut node".

Definition 5.1 *A player $i \in N$ has a **middleman position** in the network $g \in \mathbb{G}^N$ if there exists some set of links $h^* \subset L_i(g)$ under the control of player i in g such that there are at least two distinct players $j_1, j_2 \in N \setminus \{i\}$ who are connected in g and who are not connected in $g \setminus h^*$. A player with a middleman position in a network g is denoted as a **middleman** in g. The set of middlemen in the network g is denoted by $M(g) \subset N$.*

It is clear from the definition that a middleman in a network has a critical position in the sense that she can break up communication within the network between other players by deleting a well-chosen subset of her own links. A subset $h^* \subset L_i(g)$ of links that a middleman $i \in M(g)$ can delete to break up communication within a network g is called a *critical link set* for middleman i.

The following re-statement of the definition of a middleman is given without a proof. It follows immediately from the definition of a middleman position in a network.

Remark 5.2 *Let $n \geqslant 3$ and let $g \in \mathbb{G}^N$ be some network with $\# \Gamma(g) = 1$. Now, $i \in M(g)$ if and only if player $i \in N$ controls a critical link set $h^* \subset L_i(g)$ such that exactly one of the following properties holds:*

(i) $\# C(g \setminus h^*) > \# C(g) = 1$;

(ii) $\# C(g \setminus h^*) = 1$ *and there is some player* $j \in N \setminus N_0(g)$ *such that* $j \in N_0(g \setminus h^*)$, *or*

(iii) $\# C(g \setminus h^*) = 0$ *and* $N_0(g \setminus h^*) = N$.

Remark 5.2 states that a middleman in a network can either increase the number of non-trivial components in the network by removing some critical links, or disconnect some players from the network. In the latter case, the disconnected players are always *marginal* in the sense that $\# L_i(g) = 1$. Remark 5.2(iii) discusses the case of a so-called *star* network, where player i is the center of the star. Hence, $g = \{ij \mid j \neq i\}$.

The analogue of critical link monotonicity for the case of a middleman is denoted as middleman security:

Definition 5.3 *A pair $(g, v) \in \mathbb{G}^N \times \mathbb{V}^N$ is **middleman secure** if for every component $h \in C(g)$, every middleman $i \in M(h)$, and every critical link set $h^* \subset L_i(h)$ for middleman i we have that*

$$v(h) \geqslant \sum_{i=1}^{m} v(h_i) \implies \frac{v(h)}{n(h)} \geqslant \frac{v(\widehat{h})}{n(\widehat{h})}, \tag{7}$$

where $C(h \setminus h^*) = \{h_1, h_2, \ldots, h_m\}$ and $\widehat{h} \in C(h \setminus h^*)$ such that $i \in N(\widehat{h})$.[8]

Middleman security requires that middlemen do not have any incentive to disrupt communication in a network in the case of component-wise egalitarian payoffs.

It can be shown that middleman security implies critical link monotonicity.

Proposition 5.4 *Let $v \in \mathbb{V}^N_+$ be nonnegative in the sense that $v(g) \geqslant 0$ for all $g \in \mathbb{G}^N$. If (g, v) satisfies middleman security, then (g, v) satisfies critical link monotonicity as well.*

A proof is provided in Gilles, Chakrabarti, Sarangi, and Badasyan (2004).

The next proposition states our main insight regarding the relationship between the role of middlemen and the possibility to have a network that is efficient as well as strongly pairwise stable. Middleman security indeed secures that a network can be efficient as well as strongly pairwise stable under the component-wise egalitarian allocation rule.

Proposition 5.5 *If $g \in \mathbb{G}^N$ is efficient relative to a nonnegative and component additive $v \in \mathbb{V}^N_+$, then g is strongly pairwise stable for the component-wise egalitarian allocation rule Y^{ce} if and only if (g, v) is middleman secure.*

A proof of this proposition is given in Gilles, Chakrabarti, Sarangi, and Badasyan (2004).

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[8] We emphasize again that possibly $\widehat{h} = \varnothing$.

6 Conclusion

In this paper we have shown that under the component-wise egalitarian rule there is no tension between strong pairwise stability and efficiency only for middlemen secure networks. This focusses our attention on the role of middlemen in social networks. It is clear that middleman positions give occupants widespread control over the functioning of the network.

Future research has to address the profound questions that are raised by middleman positions in networks. Kalai, Postlewaite, and Roberts (1978) already investigated the consequences of middleman positions on core allocations in simple network-based exchange economies. They arrived at some surprising insights, that have great affinity with the main result from our analysis. Further research is called for to explore these profound and interesting questions.

Furthermore, our main result raises the question whether there are allocations rules other than the component-wise egalitarian allocation rule Y^{ce} that have similar properties. Since Y^{ce} is the unique allocation rule that is component additive as well as egalitarian, it combines in a unique fashion the individual incentives to pursue efficiency. Therefore, we conjecture that identifying other obvious candidates with similar properties is quite unlikely.

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