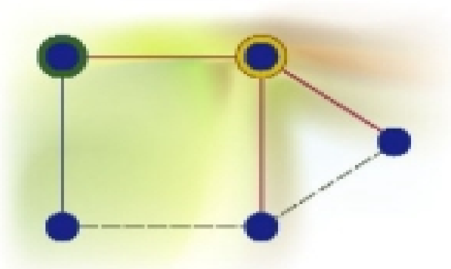
Middlemen in P2P Networks: *Stability and Efficiency*



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Motivation

- **<u>Key insight</u>**: There may be a tension between efficient and stable networks.

  -Jackson & Wolinsky (1996)

- What is the role of middlemen in attaining efficiency in stable networks?

- We analyze this using a stability concept called ***Strong Pairwise Stability***.

Model Setup

- Player set: $N=\{1, \ldots, n\}$.
- A link between players *i* and *j* exists when $g_{ij} = g_{ji} = 1$.
- Links creation requires consent but they can be broken unilaterally.

Value Functions

- Value of a graph: collective network benefits
  $V: G^N \rightarrow \mathbb{R}$ and $v(g) = \Sigma\, u_i(g)$

- A value function is component additive if v(g) is the sum of the values generated in each component.

- A value function is anonymous if the network benefits v(g) depend only on the topology of the network.

Allocation Rules

- Allocation rules specify how “value” is allocated amongst the members of g.

$$Y: G^N \times V^N \rightarrow \mathbb{R}$$

- An allocation rule is balanced if

$$v(g) = \Sigma\, Y_i(g,v)$$

for all g and v.

Allocation Rules

- An allocation rule is anonymous if the payoff of a player depends only on their position in the network.

- An allocation rule is component balanced if $\Sigma_{i \in h} Y_i(g,v) = v(h)$ for every g and $h \in C(g)$ and for every component additive v.

Allocation Rules

- Component additive + Component balance
  $\Rightarrow$ *Isolated players get a payoff of zero*.

- Component-wise egalitarian rule (Y^{ce}): The value generated in a component is split equally among members of that component.

- Y^{ce} is the unique allocation rule that is component balanced and assigns equal payoffs to all players in the same component.

Stability and Efficiency

- Link deletion proof (LDP): No player wishes to delete a single link.
- Strong link deletion proof (SLDP): No player wishes to delete a subset of their links.
- A network g is link addition proof (LAP) if for all pairs ij it holds that $Y_i(g+ij,v) > Y_i(g,v)$ implies that $Y_j(g+ij,v) < Y_j(g,v)$.

Stability and Efficiency

- LDP + LAP = Pairwise stability
- SLDP + LAP = Strong pairwise stability (SPS)
- A network g is said to be efficient with respect to the value function v if $v(g) \geq v(g')$ for all $g' \subset g_N$.

Why SPS?

- Link deletion is an unilateral act – so why restrict to deleting one at a time?

- It is an intermediate notion of link-based stability.

- Shifts emphasis from links to players: *ideal for studying middlemen!*

Additional Properties of SPS

- Players need not be stuck in “bad company” – they can isolate themselves from the network.

- Let v be a component additive value function and Y a component balanced allocation rule. Then under SPS payoffs are always bounded.

- Every pairwise stable equilibrium in the symmetric connections model is strongly pairwise stable.

Relation to earlier work

- The pair (g,v) satisfies critical link monotonicity if for any critical link ij and two associated components h_1 and h_2 of h-ij we have that

  $$v(h) \geq v(h_1)+v(h_2)$$

  implies that the per capita allocation in h is at least as great as the per capita allocation h_1 in and $h_{2.}$

Relation to earlier work

- If g is efficient relative to a component additive v, then g is pairwise stable for Y^{ce} relative to v iff (g,v) satisfies critical link monotonicity. (Jackson & Wolinsiky (1996))

- Critical link monotonicity is not adequate for strong pairwise stability!

Networks with Middlemen

- A player has a middlemen position in the network g if there exists some set of links $h^* \subset L_i(g)$ such that there are at least two distinct players j_1 and $j_2 \neq i$ who are connected in g but not in $g \backslash h^*$.

- A player with a middleman position in g is called a middleman. Middlemen have positional power in the network.

Middlemen Security

- A pair (g,v) is middleman secure if for every component $h \in C(g)$, every middleman i, and every critical link set h* we have that

  $$v(h) \geq \Sigma\, v(h_i)$$

  implies that per capita allocation in h is at least as great as the per capita allocation in the component containing the middleman i.

Networks with Middlemen

- Middleman security $\Rightarrow$ critical link monotonicity.
- If g is efficient relative to a nonnegative and component additive v, then g is strongly pairwise stable for Y^{ce} iff (g,v) is middlemen secure.

Further…

- A network is middlemen-free iff it is middlemen secure and they have similar properties.
- There is an overlap between regular networks and middlemen-free networks.
- Future research to allow for individualistic payoffs instead of just collective network benefits.